CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE ENGINEERING RESEARCH LABORATORY

WIND AND MICROTREMOR INDUCED VIBRATIONS OF A TWENTY-TWO STORY STEEL FRAME BUILDING

BY

MIHAILO D. TRIFUNAC

A REPORT ON RESEARCH CONDUCTED UNDER A GRANT FROM THE NATIONAL SCIENCE FOUNDATION

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ABSTRACT

A method of structural testing using wind and micro-tremor induced vibrations is outlined for a twenty-two story steel frame building. The results for the translational and torsional frequencies agree well with the independent determinations of the frequencies and modes obtained from vibration generator resonance tests of the same building. It is concluded that the method of structural testing based on the ambient induced vibrations can give good estimates of the frequencies and modes of vibration of building structures.

INTRODUCTION

A method of structural testing based on the measurement of vibrations caused by the wind and ground microtremor has been used occasionally in the past. The United States Coast and Geodetic Survey, for example, starting in 1936, used this method to measure fundamental periods of buildings. Recently, Crawford and Ward (Crawford and Ward, 1964; Ward and Crawford, 1966) showed that this method can be used to determine some of the frequencies and modes of vibration.

Another important approach to dynamic testing of structures is based on forced vibration tests. During these tests the structure is excited in a steady state vibration with one or more shakers which have accurate speed control (Hudson 1962). These shakers have been used successfully to test many different structures, including dams, a reservoir intake tower, atomic reactors, many different buildings, and some special structures (Jennings and Kuroiwa, 1968).

A common objective of the above-mentioned methods is to further our understanding of the dynamic properties of full scale structures. These properties include frequencies and modes of vibration and the amount of the energy dissipated by the structure. The knowledge of these properties is essential in understanding and interpreting structural response during strong earthquake ground motion, during wind excitation, and in comparisons with theoretical results.

The amplitudes of structural motion in the case of ambient vibrations depend mainly on the wind speed, since wind is usually a more effective exciting source than ground microtremors or sources in the structure. Although in both ambient and vibration generator tests the displacements are very small, the vibrator-induced vibrations are of the order of 100 times greater than the ambient vibrations. Therefore, there is a possibility that a structure might behave differently at various levels of excitation and hence the above-mentioned methods might lead to different results. It is the objective of this paper to explore this important point by comparing results from ambient and forced vibration tests performed on the same structure.

DESCRIPTION OF THE BUILDING

The San Diego Gas and Electric Building is located in the block between 1st and 2nd and Ash and A Streets in San Diego. It has a 180 ft. by 70 ft. tower six bays by two bays in plan dimension from fourth floor to the top (Fig. 1 and Fig. 2). A typical plan for fourth through seventeenth floors is given in Fig. 2; a typical story height is 13.5 ft. The foundation floor covers 300 ft. by 200 ft. area (Fig. 1). Floors A and B are used for parking and various servicings and storage space. Floor 1 is used for entrance and lobby.

The building is separated by the seismic joint into the "Low Building", which is a U-shaped structure about 180 ft. wide and 250 ft. long, and the twenty-three story "Tower". The structure for the two stories of the "Low Building" is a reinforced concrete frame. Two upper stories consist of the steel frame and concrete shear walls (Fig. 1). The tower, which is enclosed by the "Low Building" is a ductile, moment resisting steel space frame. The typical floor structure consists of a cellular steel deck with concrete topping. Because of the favorable soil conditions conventional reinforced concrete spread footings are employed. The building is designed for the earthquake loads according to the specifications for the zone 3 of the Uniform Building Code.

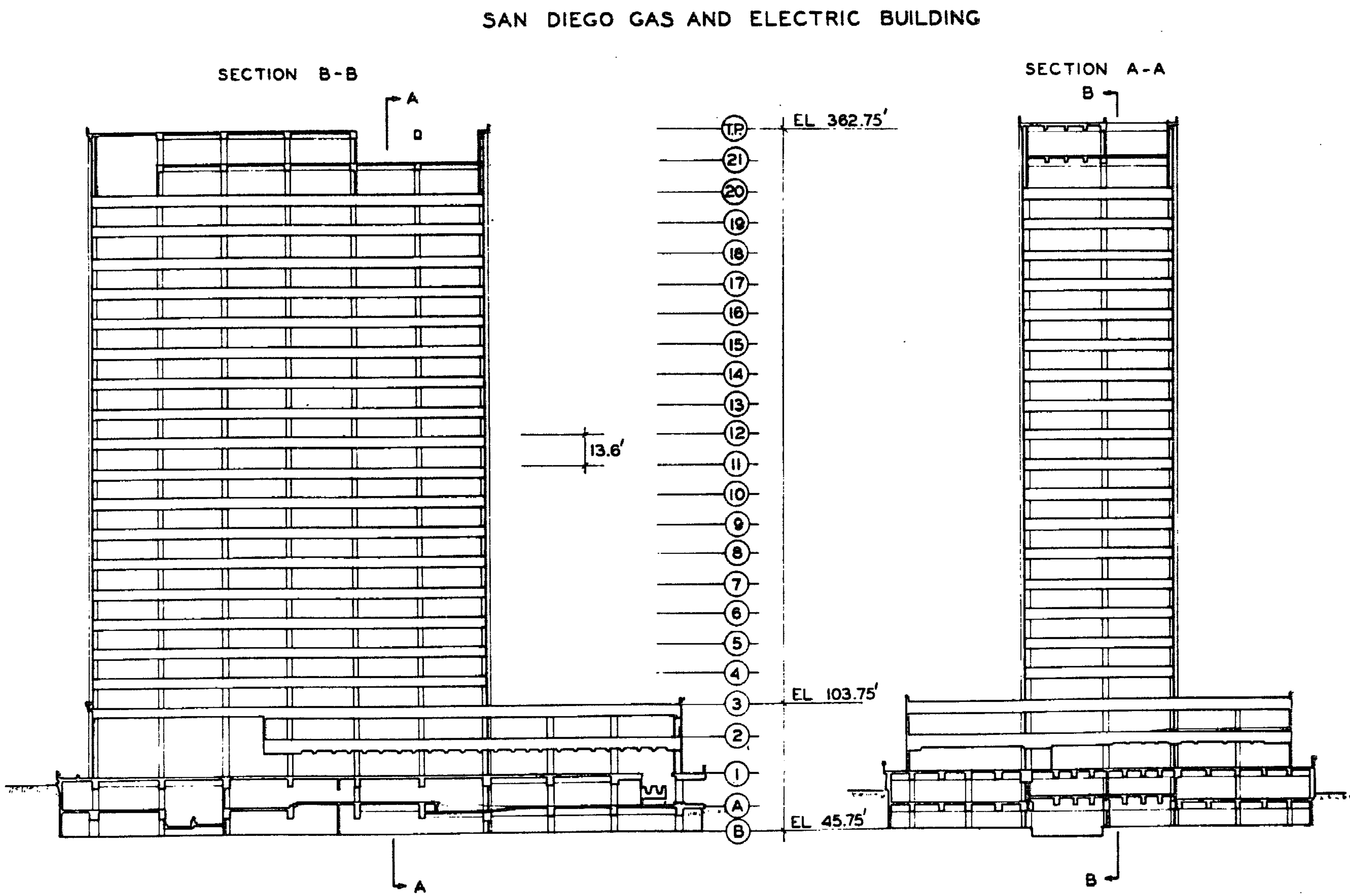

Figure 1

San Diego Gas and Electric Building,
NS and EW Sections of the Bldg.

Figure 2
Typical floor plan showing locations of ranger seismometers

DESCRIPTION OF THE MEASURING EQUIPMENT

Four Earth Sciences Ranger Seismometers were used to measure wind and microtremor vibrations of the building. These seismometers have a stationary coil with a permanent magnet as the moving seismic mass. The voltage generated in the coil is proportional to the relative velocity of the moving mass. The permanent magnet seismic mass also moves in the reversed field produced by permanent rod magnets attached to the case to provide a destabilizing force which extends the natural period of the mass and its suspension. The resulting natural period is close to 1 second. Damping is adjusted by the choice of appropriate resistance in the coil and external amplification circuits. During this experiment, the damping was set as .7 of the critical. Amplification of all frequencies greater than about 1 cps was nearly constant, and with 6 db per octave fall off for frequencies smaller than 1 cps. It may be noted here that this fall off, for frequencies smaller than about 1 cps, can be quite advantageous in recording vibrations of structures with fundamental period close to 2 seconds or longer. In this way, the low frequency modes do not dominate in the records which facilitates the study of higher modes. In such cases instead of recording displacement one may record velocity or even acceleration and further emphasize the high frequency content.

An Earth Sciences SC-201A Signal Conditioner was used to amplify and control simultaneously four outputs from Ranger Seismometers. Each data channel on the signal conditioner has amplification of up to 350,000 times and attenuation from 0 to 82 db. The power for the signal conditioner is provided by internal rechargeable batteries with nominal 25 hours of operation between charges. When used with a velocity transducer each channel can provide output voltages proportional to displacement, velocity and acceleration.

Depending on the level of building vibration during this experiment, various attenuator settings were used ranging from 40 db. to 58 db. This provided on the average about 3,000 volts/inch of the displacement. Recorded signals were usually kept within about 1 volt in amplitude which indicates that the measured displacements were of the order of 10^{-3} cm or ten microns. It is of interest to note here that during the forced excitation of the same building amplitudes were of the order of 1 mm or about 100 times greater. The level of the ambient vibration of a structure like San Diego Gas and Electric Building depends mainly on the intensity of wind. During the weekend of July 19 to 20, 1969 when measurements were conducted wind intensity was relatively low.

The voltage proportional to the relative velocity of the seismic mass was recorded by a Lockheed Electronics Model 417 Magnetic Tape Recorder. In order to have immediate visual inspection of the vibrations, the seismic signals were simultaneously recorded on two Mark 220 Brush Recorders. Each of these recorders has two channels so that all four outputs from the seismometers were displayed simultaneously.

MEASURING PROCEDURES

When measuring ambient or shaker excited building vibrations, one usually assumes that the structure can be approximated by a one-dimensional, linear, damped discrete or continuous system. In some cases (Jennings and Kuroiwa, 1968) measurements indicate that floor structures are sufficiently stiff so that the above assumption is acceptable. In the case of the San Diego Gas and Electric Building, it is assumed that structural behaviour may be approximated by a linear one-dimensional system. If this assumption is valid, then the measurement of vibration modeshapes becomes relatively simple. In order to fully understand all features of the building vibration, one would, of course, have to consider three-dimensional or at least two-dimensional models. From the point of view of the experimental measurement such an approach would in principle be as easy as the one-dimensional case, but the tests would of course involve more measurements and an increased data processing effort.

In the experimental study of building vibration which is based on the linear model, it is assumed that the resulting motions can be expressed as the superposition of modes associated with discrete frequencies. This approach then requires a simultaneous measurement of motion in a given direction at at least two different floors. It may be mentioned here that for the measurement of wind induced vibrations, it is not necessary to calibrate all instruments so that they give the same amplitudes when excited by the same base motion. It is also not necessary to find the actual amplitudes that are recorded, because all that is ever used in determining modeshapes is the relative amplitude of the same two instruments.

The measurements in the San Diego Gas and Electric Building were conducted in the following way. To obtain translational modeshapes and associated frequencies, two seismometers were permanently located on the 20th floor in the north stairwell (Fig. 1, and Fig. 2). One instrument was oriented in the north and the other in the east direction. Two other instruments oriented in the same way were located on the roof during the first measurement, then on the 20th floor, 18th floor, and were consequently displaced two floors down, with exception of 13th floor where additional records were taken. During all translational measurements listed in Table 1, the four instruments were placed in the north stairwell at the same place (Fig. 2).

To obtain more information about the torsional frequencies, three experiments were conducted at 20th, 13th, and 12th floors. In these tests, two instruments oriented E were placed in extreme NE and SW corners of the tower as indicated in Fig. 2.

For measurement of the translational modes one would ideally try to put all instruments in the center of the structural cross section or in the center of torsion. Because the center of torsion was not known, and because the floor was carpeted at the center of the building (placing an instrument on carpet would have some undesirable filtering effects for certain frequencies), it was decided to place all instruments in the north stairwell directly on the concrete slabs. This decision had also the advantage that it permitted determination of a few torsional modeshapes.

TABLE NO. 1

Sequence of Tests to Determine Modeshapes.
Instrument Locations and Orientations During 13 Tests

Test No.	Reference Location (Flr. No.)		Variable Location (Flr. No.)	
	N	E	N	E
1	20	20	roof	roof
2	20	20	20	20
3	20	20	18	18
4	20	20	16	16
5	20	20	14	14
6	20	20	13	13
7	20	20	12	12
8	20	20	10	10
9	20	20	8	8
10	20	20	6	6
11	20	20	4	4
12	20	20	2	2
13	20	20	A	A

DATA ANALYSIS

During all stages of measurement four simultaneous analog outputs were recorded on magnetic tape. Before any computer analysis this analog data had to be converted to digital form on a magnetic tape compatible with the digital computer to be used.

Through analog to digital conversion 200 discrete points per second were generated for each analog record. A typical set of four simultaneous records, about 20 seconds long, from the first test (Table 1) is shown in Fig. 3. During data conversion a scale of 10,000 per volt was adapted as indicated in Fig. 3. As may be seen in this figure, the two top records contain a high-frequency noise. This noise was close to 30 cps and higher and was present in some records during the whole experiment. We were not able to determine precisely its origin, but it seems likely that it was caused by some of the installations in the building which were not shut off during the measurement such as air conditioners, elevators, or other. At times this noise would abruptly disappear during recording and then again appear. Since all frequencies in the records that could be used lie well below 30 cps, it was decided to filter the original records with a digital low-pass filter.

Once high-frequency noise is filtered out, one does not need 200 pts per second to accurately describe the relatively smooth function which remains. Fewer points in the records considerably reduce computer time required for further data analysis, and thus permit more economical data processing. It was decided that 50 points per second would be appropriate for present purposes because this

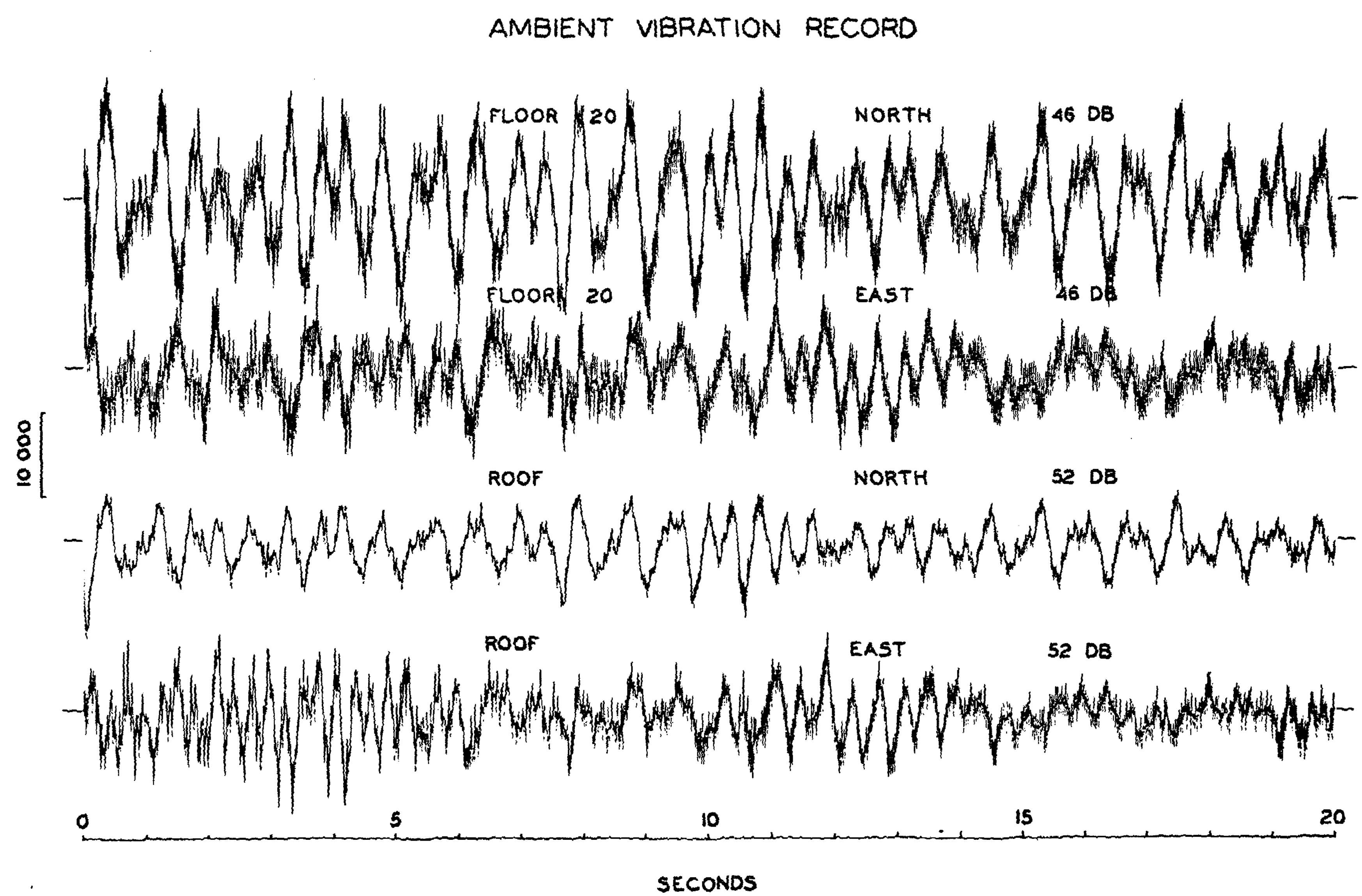

Figure 3

Typical 20 seconds sample of four simultaneous records. Smoothed record is superimposed over the original record which contains high frequency noise.

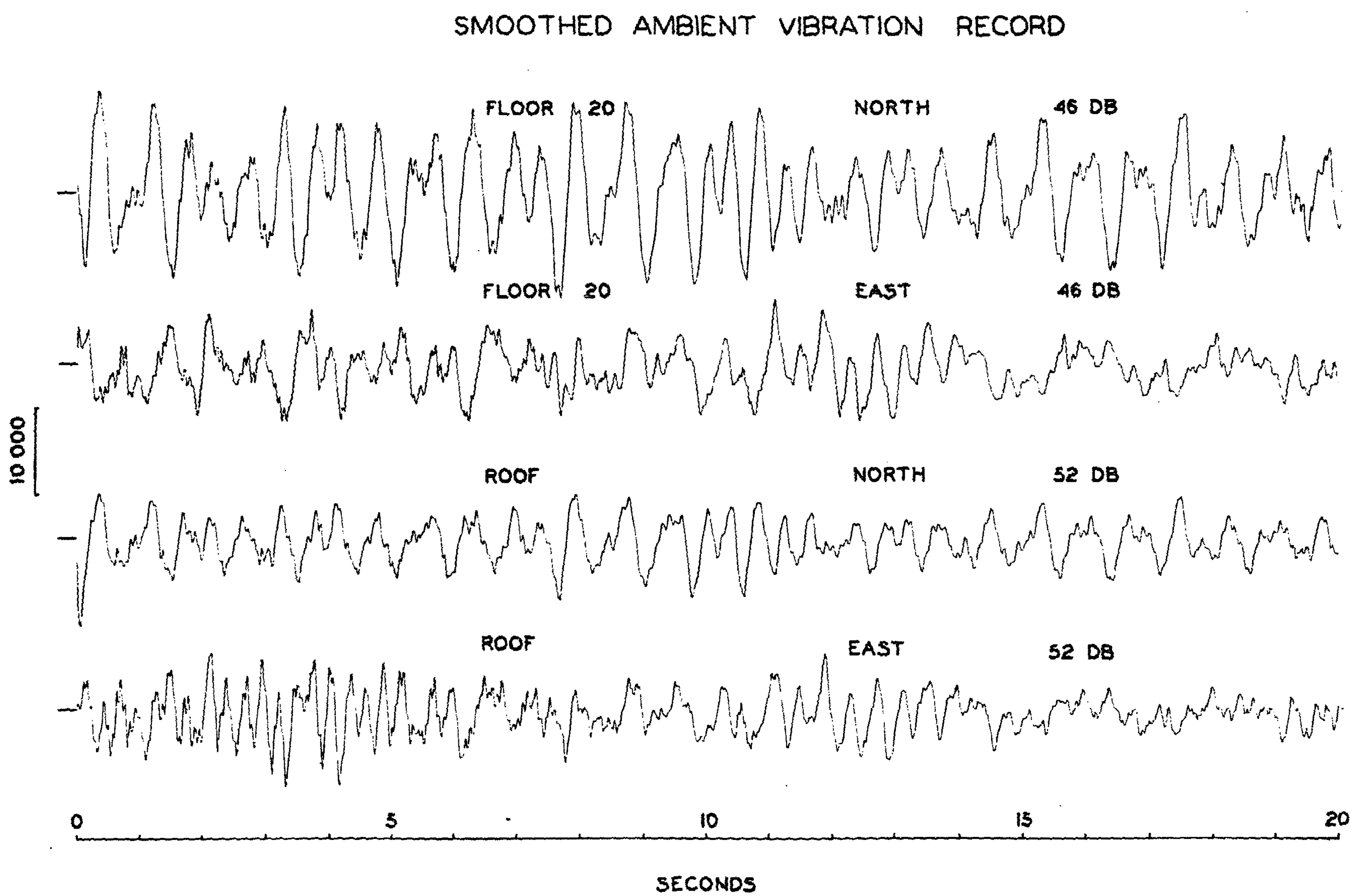

Figure 4

Typical 20 seconds sample obtained after filtering high frequency noise present in the original record (Figure 3)

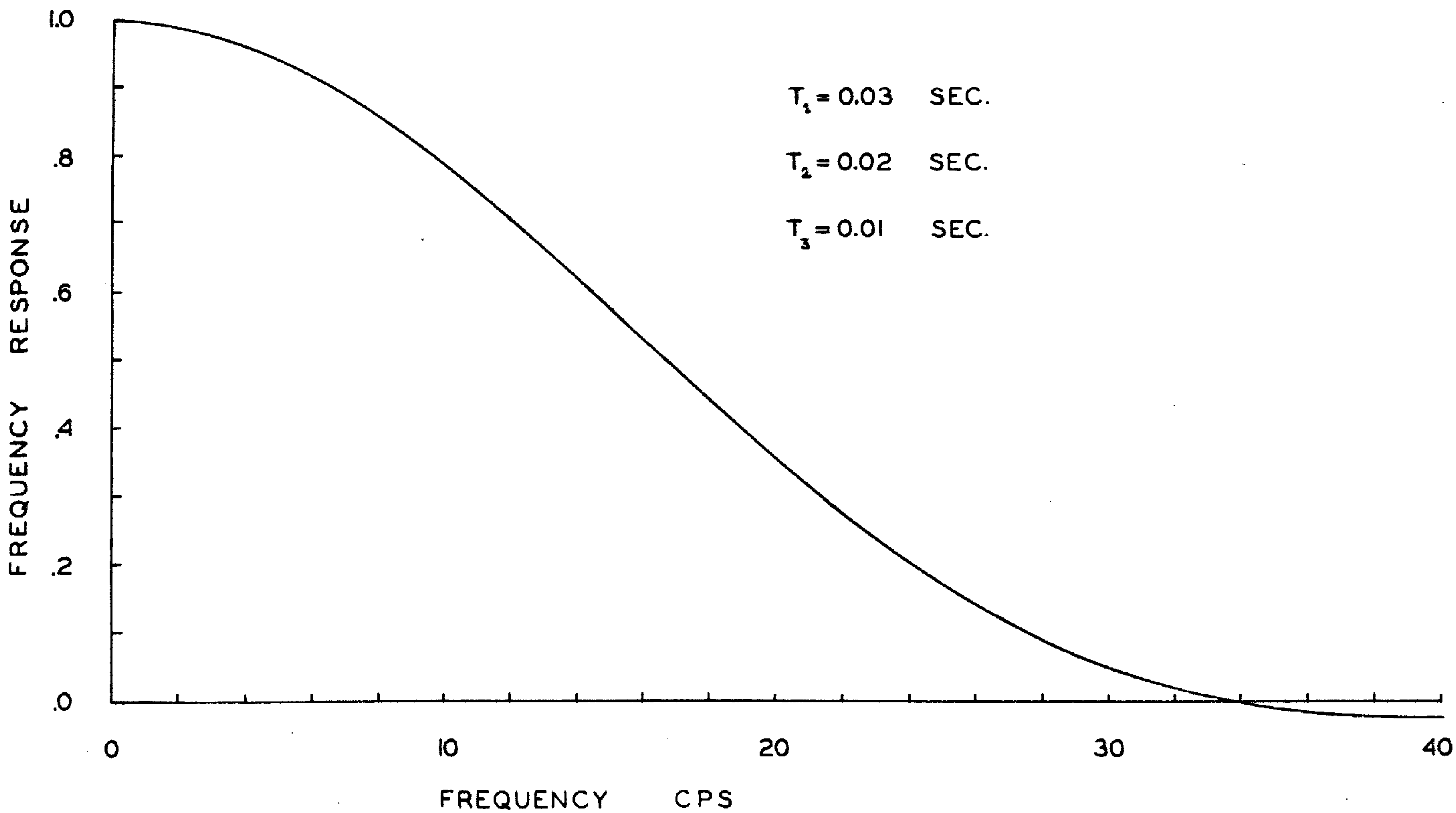

Figure 5

Frequency response function resulting from the successive digital filtering by the equally weighted running means over the time intervals $T_1 = 0.03$ sec., $T_2 = 0.02$ sec., $T_3 = 0.01$ sec.

gives a Nydquist frequency $f_n = 25$ cps which is well above all the frequencies to be studied here. The original data with 200 points per second were filtered by successive equally weighted running mean filters over time intervals of $T_1 = 0.030$ sec., $T_2 = 0.020$ sec., and $T_3 = 0.010$ sec. The overall resulting filter response function corresponding to this smoothing procedure is given in Fig. 5. As it may be seen, frequencies above about 30 cps are essentially filtered out. After smoothing every fifth point is retained from the original 200 pts per second array. The typical record shown in Fig. 3 appears as in Fig. 4 after smoothing. It consists of a total of $2^{10} = 1024$ data points or 20.46 seconds for each trace.

During the field measurement on the average about 1 minute or more of the vibration was recorded on the tape for each experiment. From each of these records one typical interval 20.46 seconds long was selected for the analysis.

The Fourier amplitude spectrum for each 20.46 seconds long record was obtained by using the Cooley-Tukey algorithm (Cooley and Tukey, 1965) which requires 2^M equally spaced data where M is an integer. For each test (Table 1) four Fourier amplitude spectra and phase spectra were computed. In addition, one needs to find the ratio of the spectra from records coming from columns 1 and 3 and columns 2 and 4 in Table 1. Before these ratios are computed, the Fourier spectra are smoothed so that the spectrum at i^{th} point f_i is replaced by f_i^{SM} as follows:

$$f_i^{SM} = \frac{1}{2} f_i + \frac{1}{4}(f_{i+1} + f_{i-1})$$

Typical smoothed Fourier amplitude spectra for test No. 1 (Table 1) are given in Figs. 6 and 7.

By taking the ratio of the Fourier amplitude spectrum at the 20th floor and the spectrum at some other floor, one obtains an amplitude proportional to the modeshape amplitude at that floor for a given frequency of vibration. As will be seen, this simple approach gives results which compare favorably with independently obtained modeshapes from shaker vibrations of the same building (Jennings and Hoerner, personal communication). When choosing the frequency at which the ratios are to be taken in order to determine modeshapes, it is not always desirable to pick exactly the peak values of spectra for the following reasons. It often happens that two translational or a translational and a torsional mode are at a nearly the same frequency, and the problem of separating the modes may be troublesome. In such cases, a point away from the peak is selected in such a way that it is as far as possible from the other frequency in question.

Another difficulty which appeared, and is also likely to occur elsewhere, is that the spectral peaks seem not to be at the same frequencies for various floor levels. Such phenomena were also reported in the case of the shaker induced vibrations (Jennings and Kuroiwa, 1968). One possible explanation for this is that various modes of vibration are excited in different amounts from one experiment to the other, because of the variable wind speed and direction. Thus, if there are two modes closely spaced in frequency space, superposition of their Fourier amplitude spectra might lead to the shifting of the apparent peak. For this reason, it may be erroneous

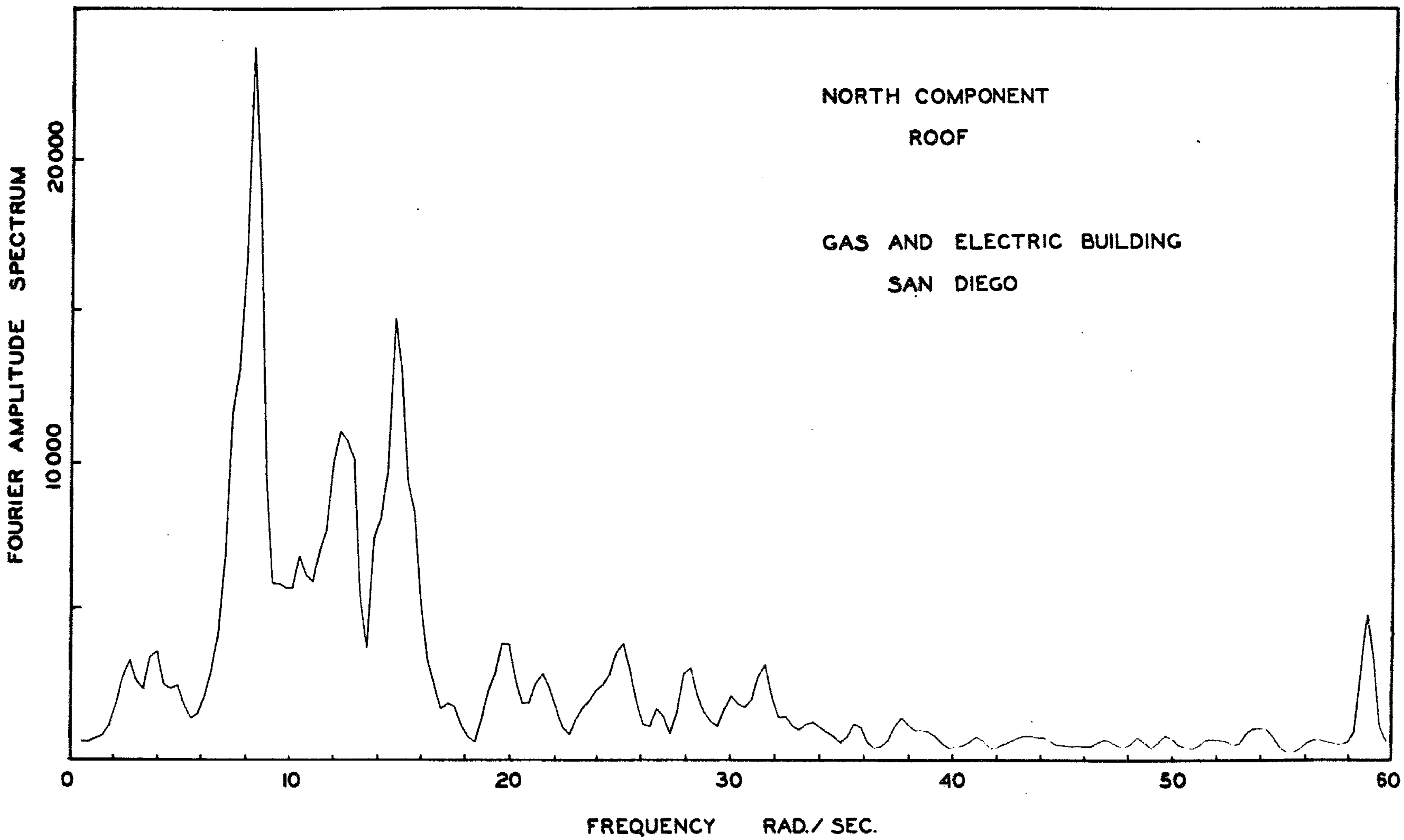

Figure 6

Fourier amplitude spectrum of the smoothed north component of the velocity recorded on the roof (Fig. 4). The scale for the Fourier amplitudes is same as indicated in Fig. 4.

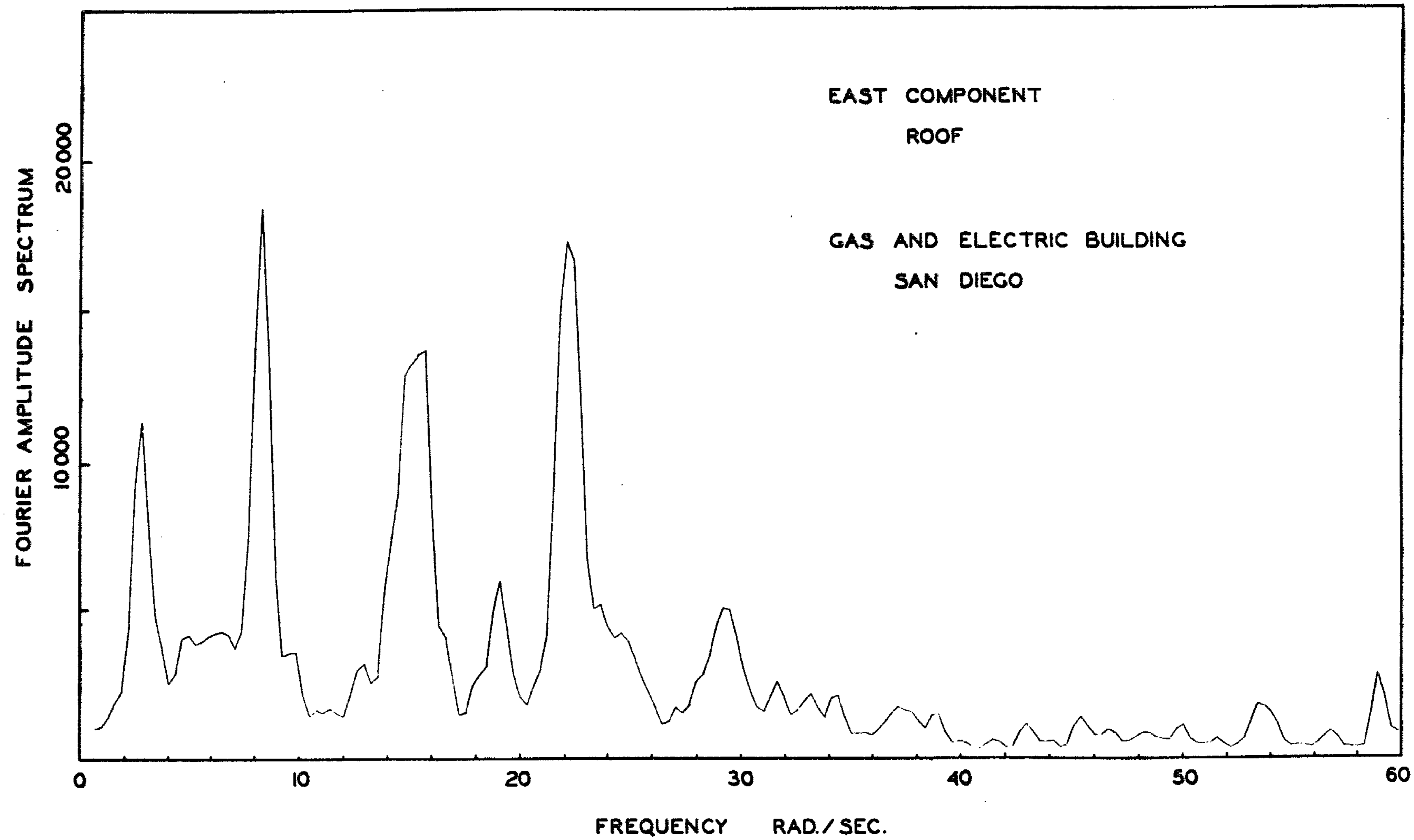

Figure 7

Fourier amplitude spectrum of the smoothed east component of the velocity recorded on the roof (Fig. 4). The scale for the Fourier amplitudes is same as indicated in Fig. 4.

to determine characteristic frequencies from one or two spectrum peaks only. The distribution of spectral peaks versus frequencies from all 13 tests listed in Table 1 was plotted and the frequencies of natural vibration were picked to correspond with the points about which the data clustered.

Since Fourier amplitude spectra give only the modulus of the amplitude, phase spectra are used to determine the positive (in phase) or negative (180 degrees out of phase) sign of the mode amplitude.

FREQUENCIES AND MODES OF VIBRATION

Natural frequencies of vibration were determined, as has already been mentioned, by considering the distribution of all peaks in the Fourier spectra for 13 tests. For this reason the peaks in a typical Fourier amplitude spectrum (Fig. 6 and 7) may not exactly agree with the values given in Table 2. As may be seen in Figs. 6 and 7 the recorded velocity did not have big spectral amplitudes above 30 rad./sec. and this gave a limit above which it was very difficult to separate structural motion from other recorded noise.

The frequencies for six EW and NS translational modes and for five torsional modes are given in Table 2. The frequency of the sixth mode in EW direction differs by 18 percent when compared with the shaker experiment. Although this frequency is well defined in the ambient vibration test, it was not possible to determine whether it represents the sixth EW or torsional vibration. For simplicity this frequency is throughout this work listed under the sixth EW translational vibration.

As it may be seen from Table 2, except for the fifth and sixth NS modes, frequencies measured from the wind excited motions are always the same or higher than those determined by the shaker experiment. In this respect, it may be interesting to note that in nearly all forced vibration tests there have been observed monotonic increases of resonant frequency for decreasing amplitude of vibration. For the range of their experiments, Jennings and Kuroiwa (1968) found that a decrease of resonance amplitude by a factor close to 8

TABLE 2

Mode No. i	NS Translational Freq. $2\pi f_i$ rad./sec.			EW Translational Freq. $2\pi f_i$ rad./sec.			TORSIONAL Torsional Freq. $2\pi f_i$ rad./sec.		
	Ambient Experiment	Shaker* Experiment	Ratio f_i/f_1 from Ambient Experiment	Ambient Experiment	Shaker* Experiment	Ratio f_i/f_1 from Ambient Experiment	Ambient Experiment	Shaker* Experiment	Ratio f_i/f_1 from Ambient Experiment
1	2.7	2.5	1.0	2.5	2.4	1.0	2.7	2.6	1.0
2	7.5	7.0	2.8	8.2	7.5	3.3	8.1	7.8	3.0
3	12.5	12.5	4.6	15.4	14.3	6.2	14.6	13.5	5.4
4	19.0	18.9	7.0	22.2	21.4	8.9	21.3	21.1	7.9
5	25.3	26.8	9.4	29.1	26.9	11.7	29.0	28.1	10.8
6	31.7	32.1	11.8	37.8	31.0	15.1			

* Jennings and Hoerner, personal communication

results in an increase of resonant frequency by about 2.5 percent. If we disregard sixth EW mode we find from Table 2 that on the average frequencies determined by the wind and ambient excitation are about 4 percent higher than those obtained by the shaker experiment. As was already mentioned, amplitudes during shaker experiment were about 100 times greater than during the ambient test. This suggests the important conclusion that the ambient vibration tests, which are based on the recording of the displacements many times smaller than forced vibration experiments, lead to the frequency and mode determinations which are in agreement with shaker results.

It is informative to find the ratios of the determined higher frequencies with respect to the fundamental one. These ratios indicate a type of overall structural response and are given in Table 2 and Fig. 8. From Fig. 8 it may be concluded that the building vibration in NS direction (Fib. 1, Fig. 2) is predominantly of the shear type because the determined frequency ratios follow closely ratios 1, 3, 5, 7, 9, 11 .... for the uniform shear beam. The results indicate that bending effects are more predominant in the EW direction.

Modeshapes determined from the ratio of the Fourier amplitude spectra are given in tabulated form in Table 3 for NS direction, Table 4 for EW and Table 5 for torsional vibrations. All determined modes of vibration are also given in Figs. 9 through 11. Torsional NS and EW fundamental frequencies are very closely spaced and the exact separation of modes is not possible from this set of data. For this reason, first torsional and EW modeshapes are taken with same amplitudes (Table 5).

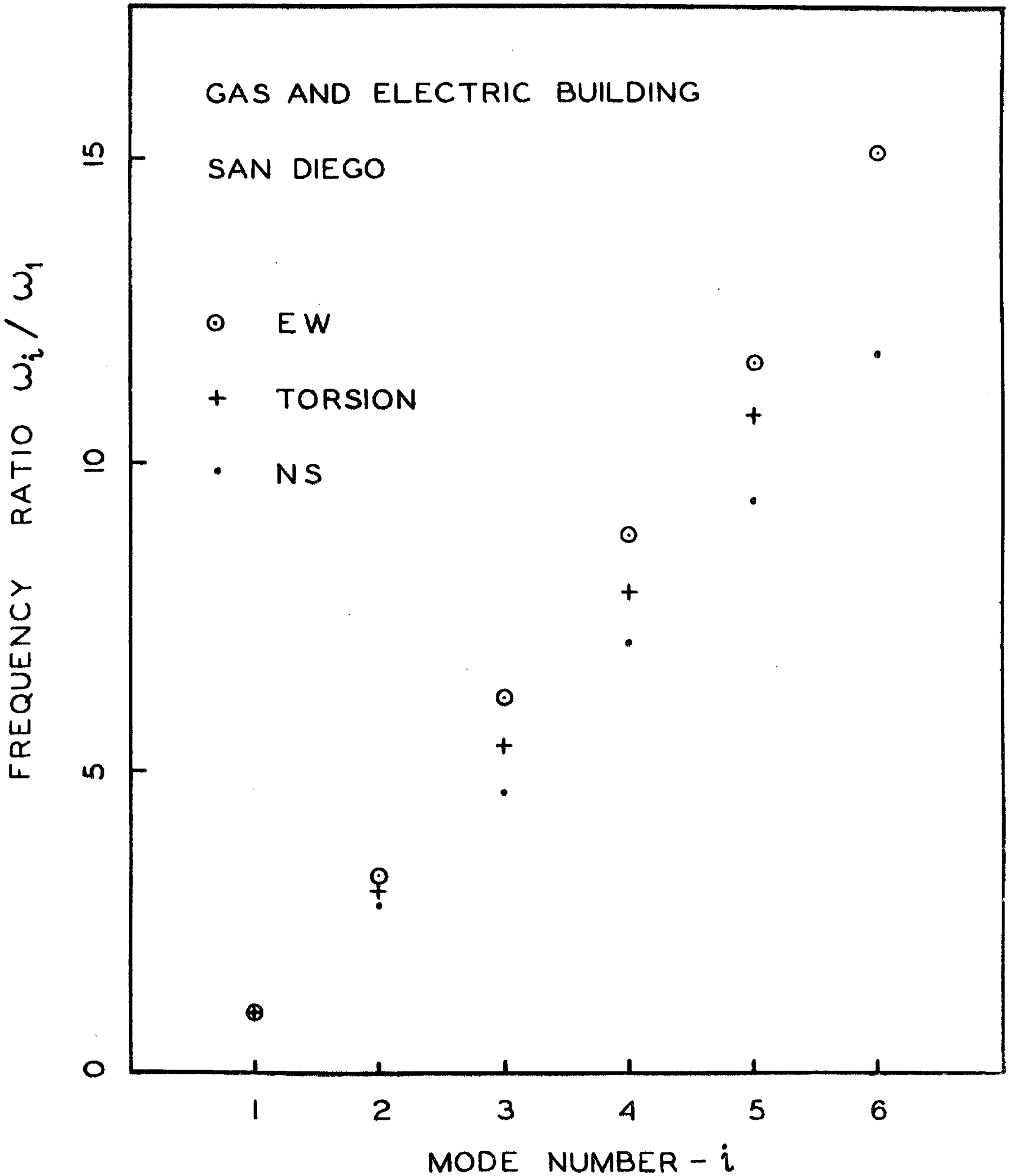

Figure 8

The ratios of the determined higher and the fundamental frequencies for the six modes

TABLE 3

NS MODESHAPES

	1	2	3	4	5	6
R	1.00	.73	.47	.51	1.00	.61
20	.96	.71	.42	.33	.31	.16
18	.93	.40	.16	-.39	-.59	-.60
16	.82	-.17	-.41	-.70	-.43	-.21
14	.76	-.54	-.69	-.29	.51	1.00
13	.66	-.66	-.75	.40	.40	.68
12	.61	-.84	-.43	1.00	.15	.35
10	.52	-1.00	.13	.90	-.32	-.41
8	.36	-.87	.89	.34	-.29	-.53
6	.24	-.53	1.00	-.52	.25	.60
4	.10	-.38	.53	-.71	.46	.94
2	.04	-.11	.17	-.23	.38	-.64
A	~0	-.02	.13	-.11	.07	-.40

TABLE 4

EW MODESHAPES

	1	2	3	4	5	6
R	1.00	.80	.78	.71	.97	1.00
20	.93	.69	.45	.20	.30	.46
18	.82	.32	-.32	-.63	-.88	-.56
16	.77	-.21	-.96	-1.00	-.32	-.35
14	.70	-.48	-1.00	-.25	.67	.43
13	.65	-.70	-.78	.23	.84	.65
12	.57	-.79	-.60	.72	.44	.63
10	.47	-.92	.20	.78	-.43	-.77
8	.36	-1.00	.73	.26	-1.00	-.56
6	.21	-.73	.78	-.46	-.27	.88
4	.12	-.45	.50	-.65	.65	.43
2	.05	-.17	.34	-.55	.34	-.50
A	.01	-.03	.04	-.10	.12	-.17

TABLE 5

TORSIONAL MODESHAPES

	1*	2	3	4	5	6
R	1.00	.80	.63	.55	.84	
20	.93	.72	.56	.27	.34	
18	.82	.35	-.34	-.41	-.59	
16	.77	-.14	-.80	-.62	-.14	
14	.70	-.58	-.97	-.31	.61	
13	.65	-.78	-.85	.53	.83	
12	.57	-.88	-.62	.83	.53	
10	.47	-.99	.14	1.00	-.42	
8	.36	-1.00	.72	.38	-1.00	
6	.21	-.65	1.00	-.43	-.18	
4	.12	-.37	.67	-.88	.50	
2	.05	-.10	.20	-.21	.67	
A	.01	-.02	.11	-.06	.18	

* Same as EW mode

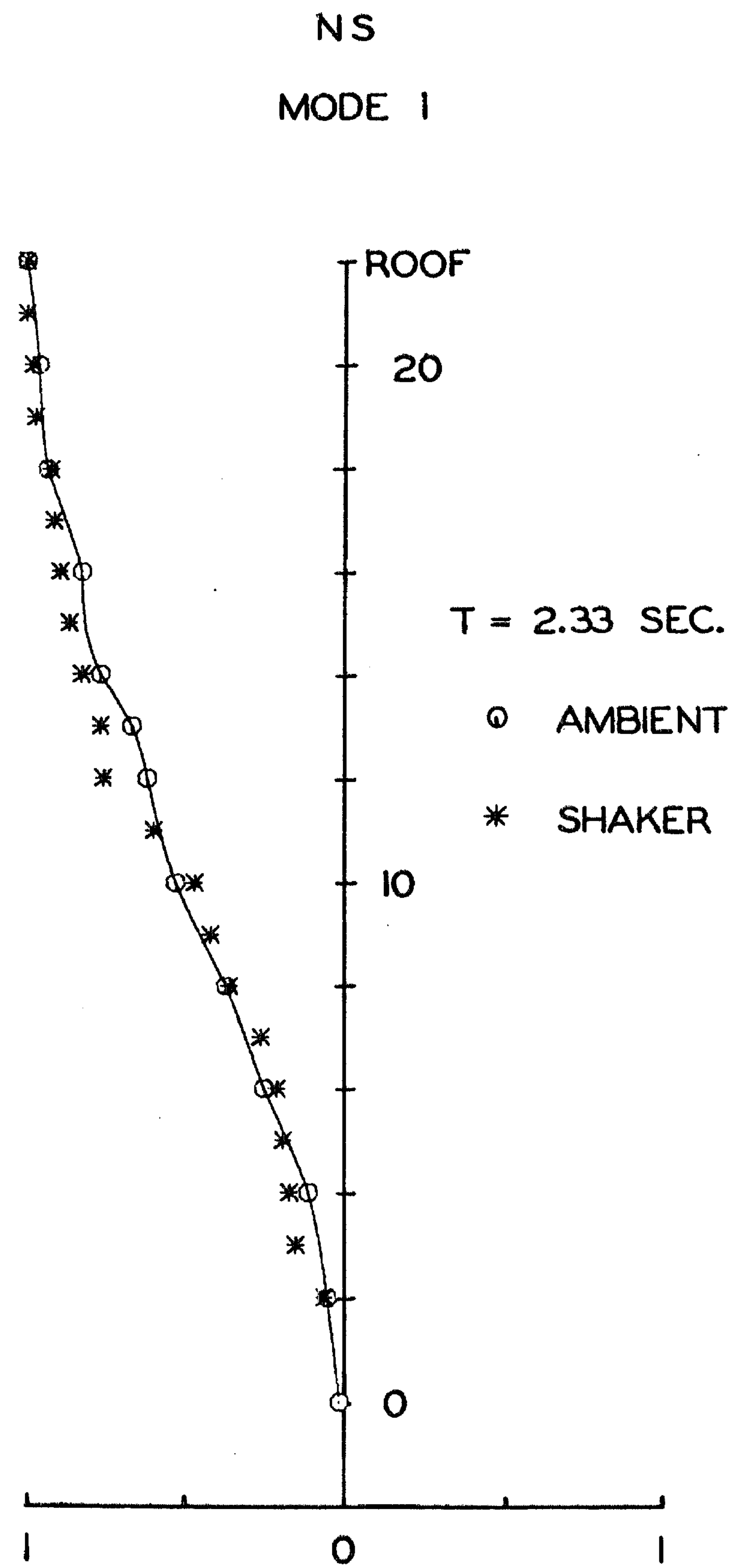

Figure 9a
First NS Mode

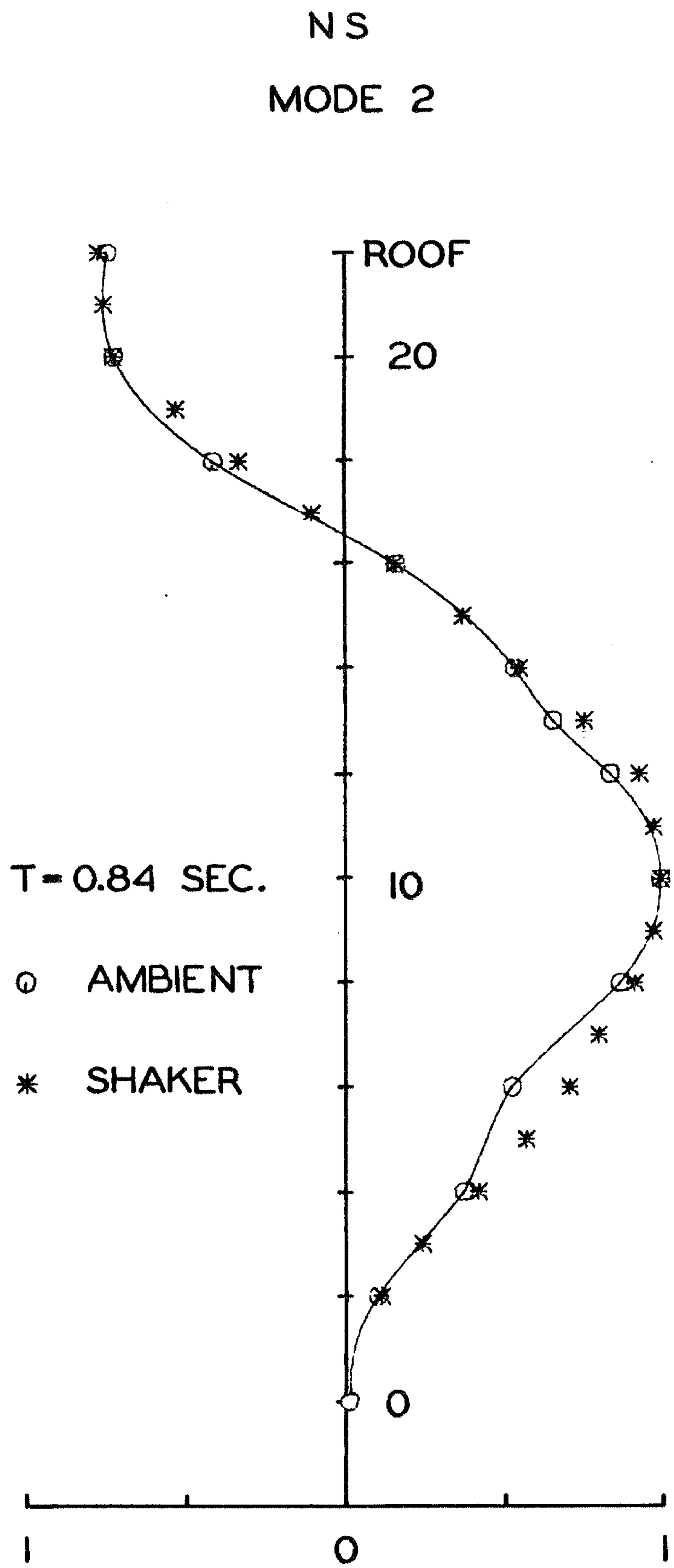

Figure 9b
Second NS Mode

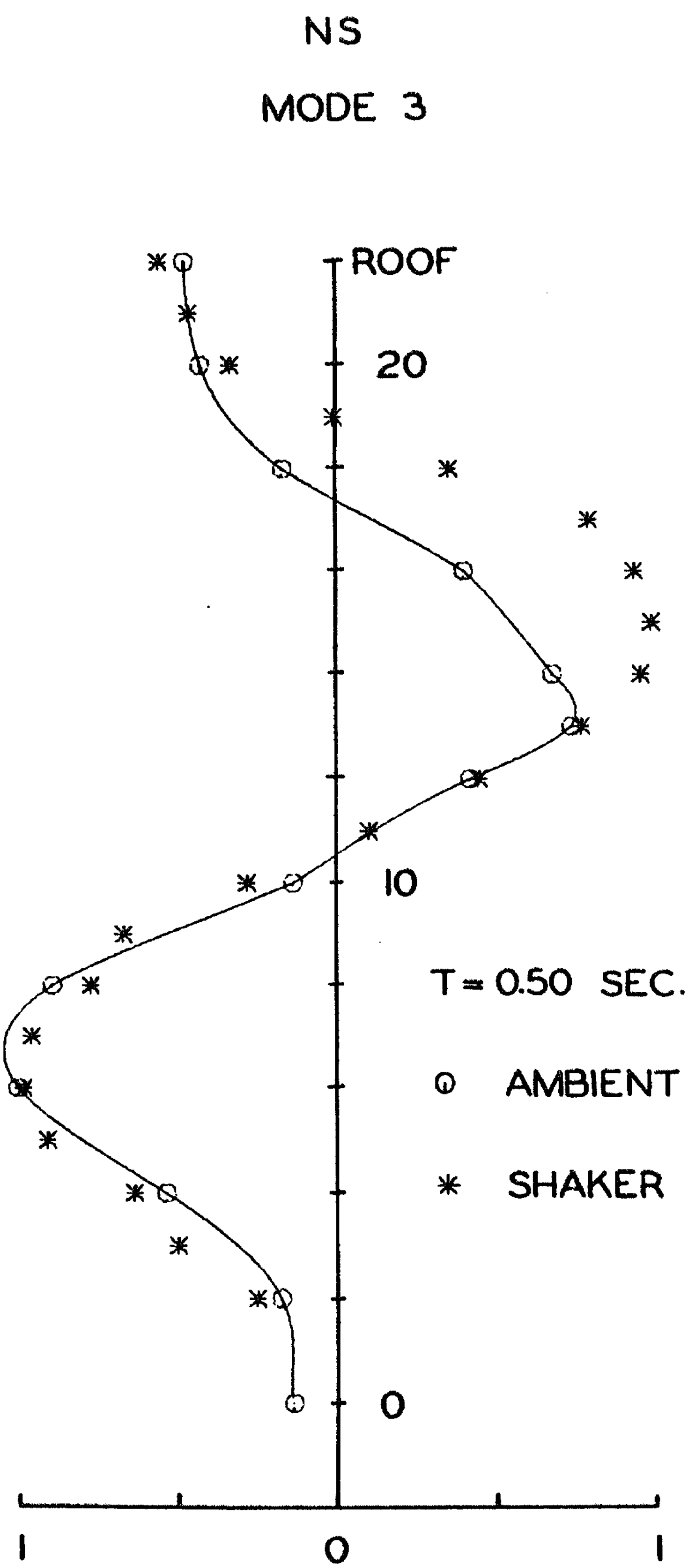

Figure 9c
Third NS Mode

NS

MODE 4

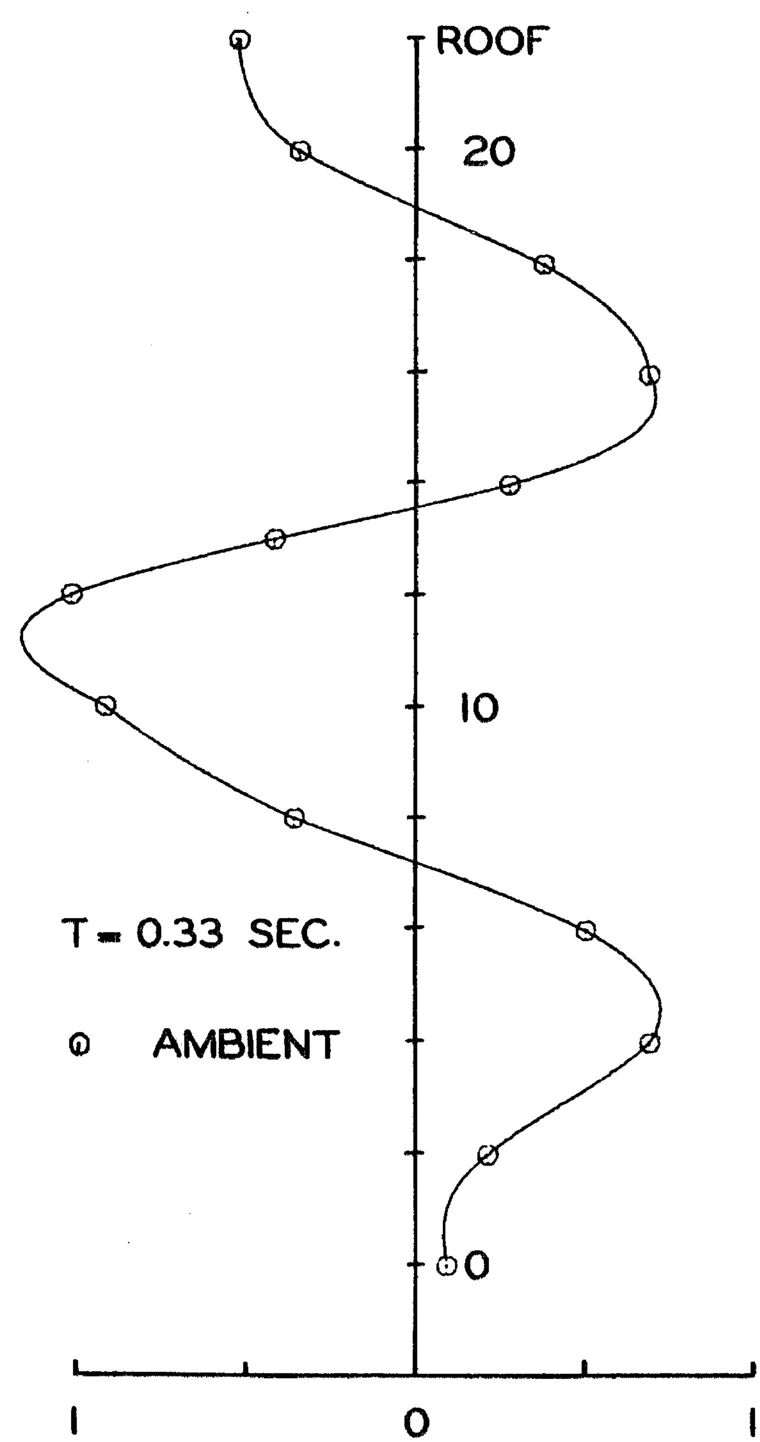

Figure 9d
Fourth NS Mode

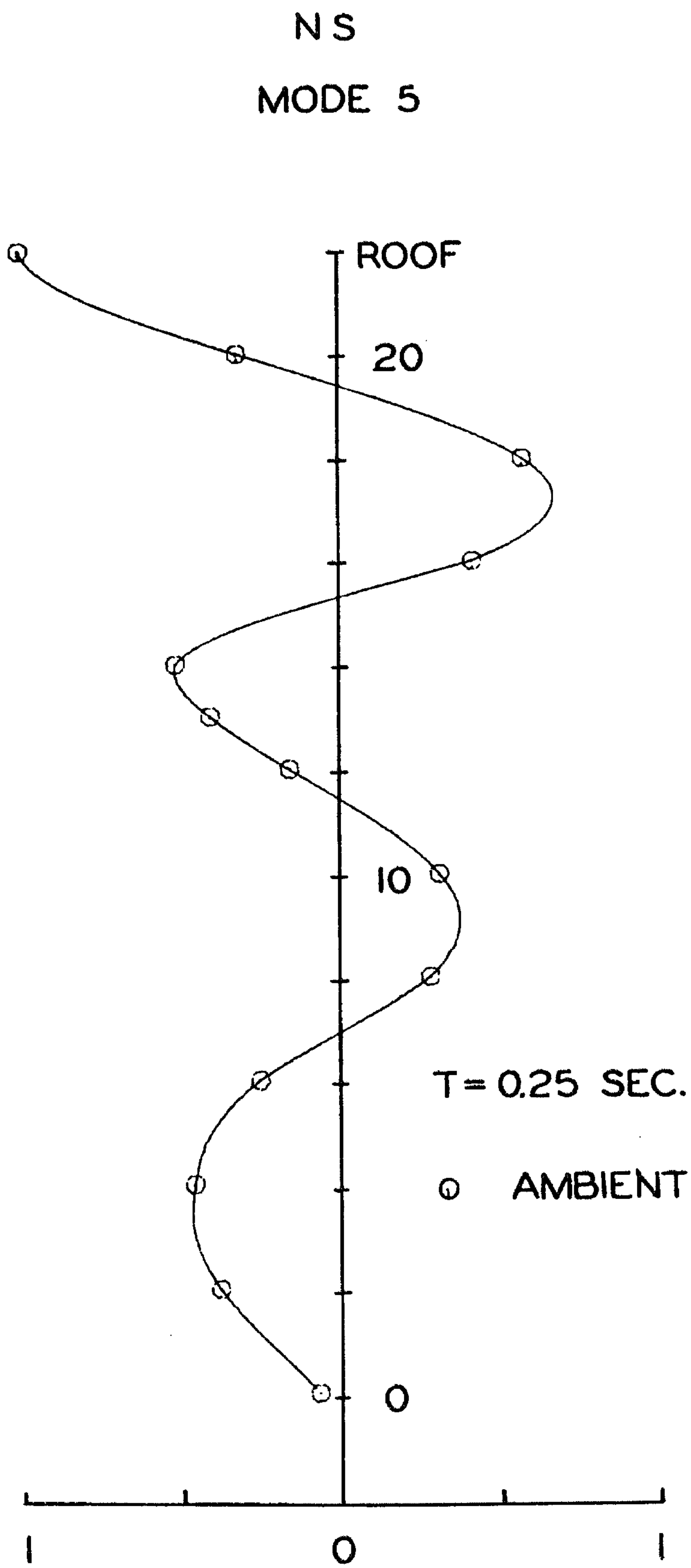

Figure 9e
Fifth NS Mode

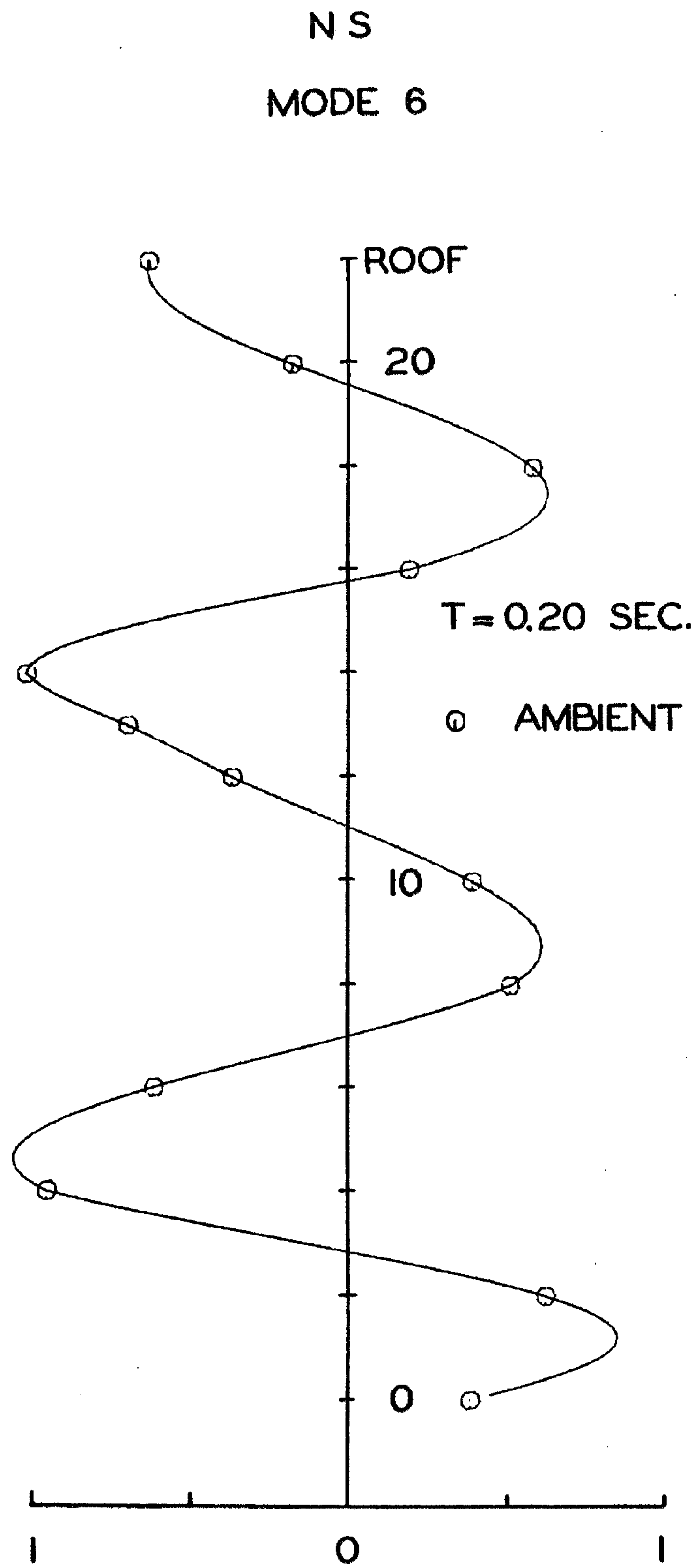

Figure 9f
Sixth NS Mode

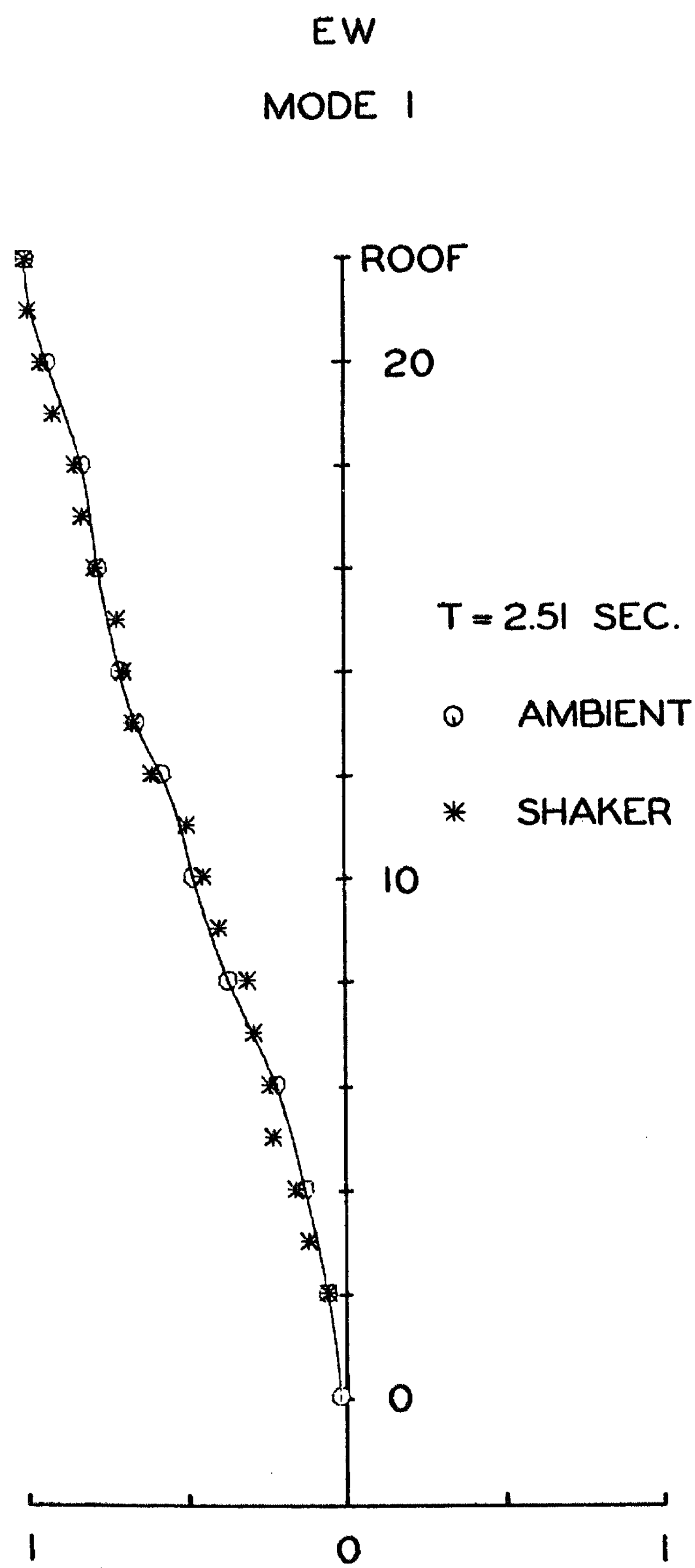

Figure 10a
First EW Mode

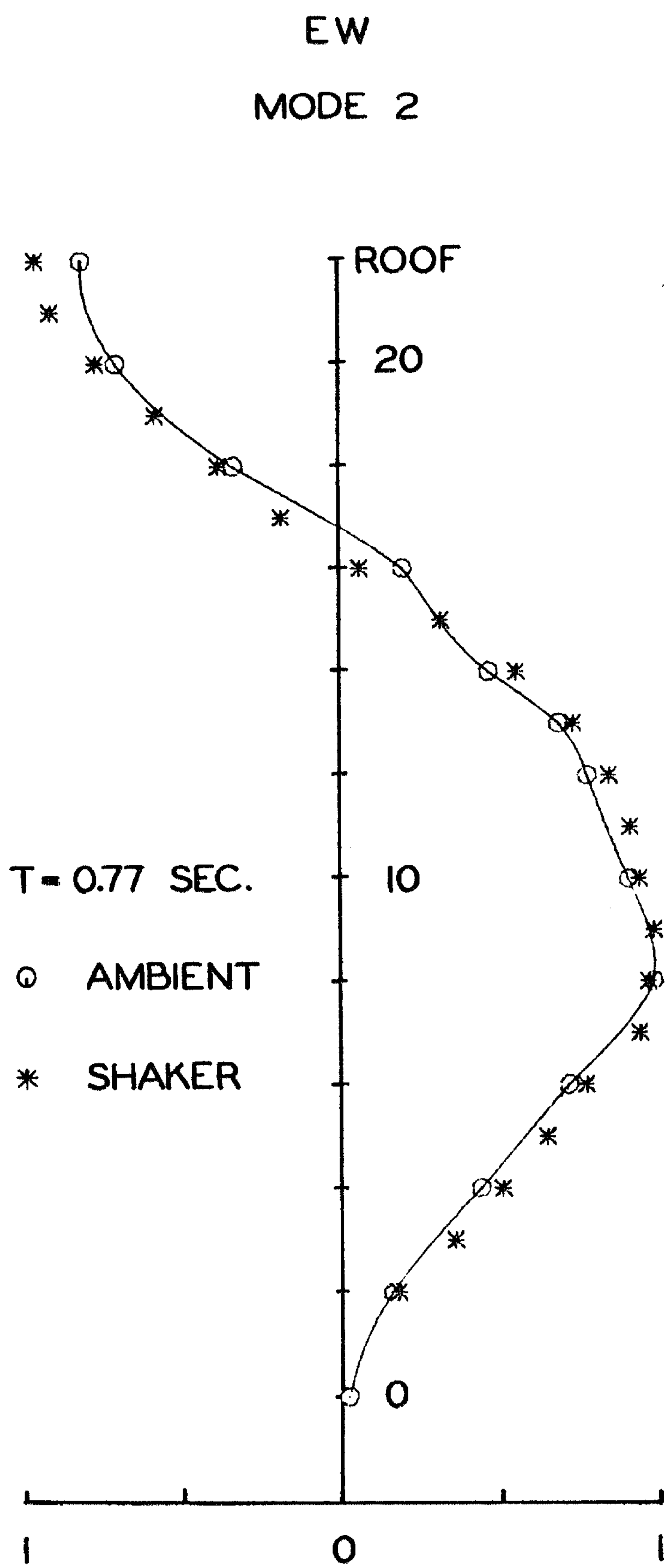

Figure 10b
Second EW Mode

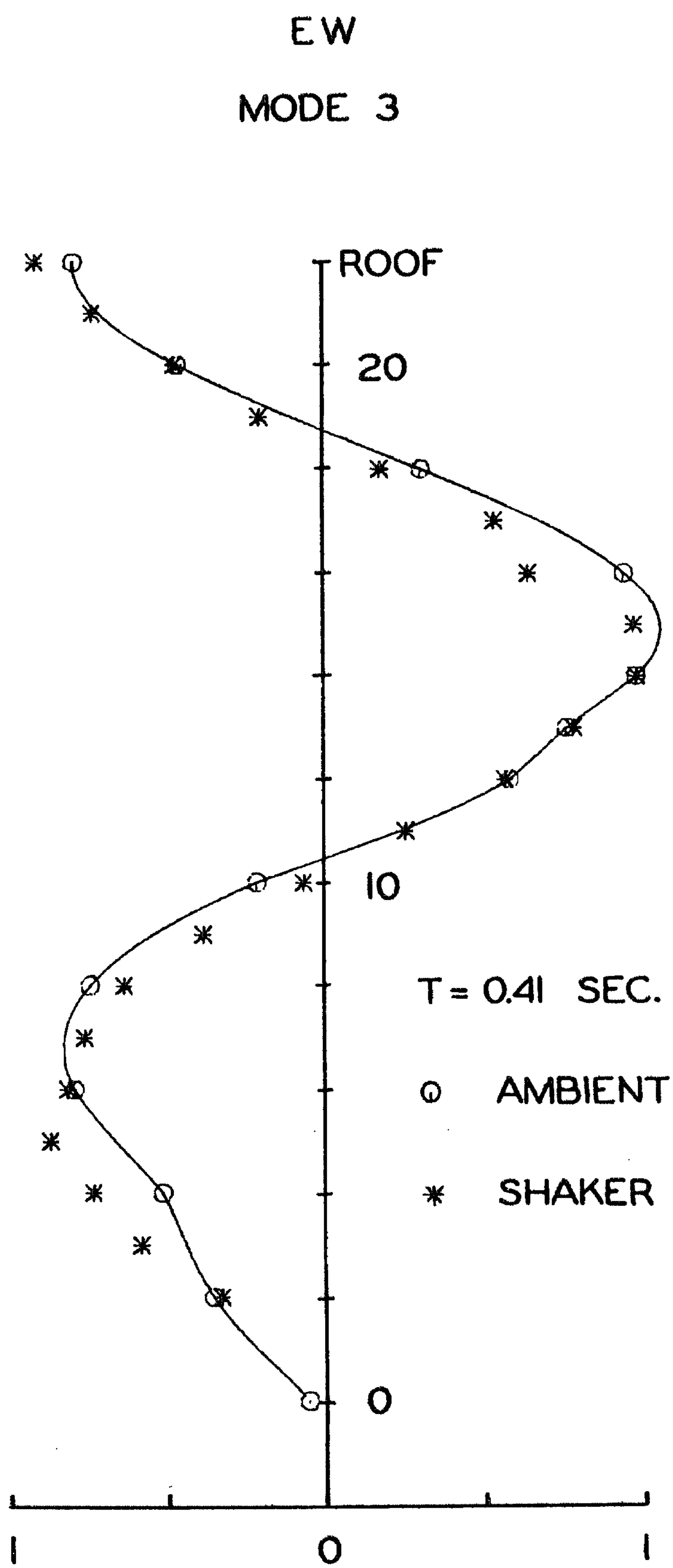

Figure 10c
Third EW Mode

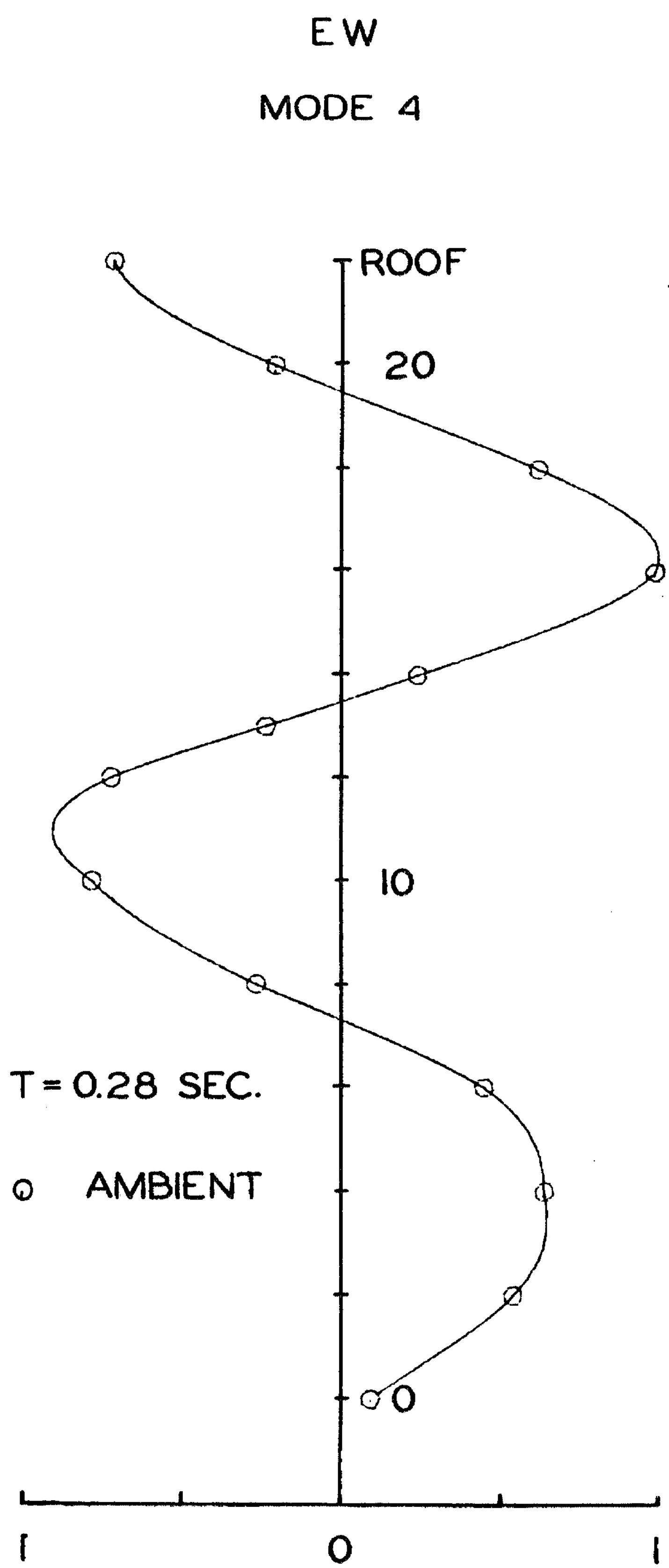

Figure 10d
Fourth EW Mode

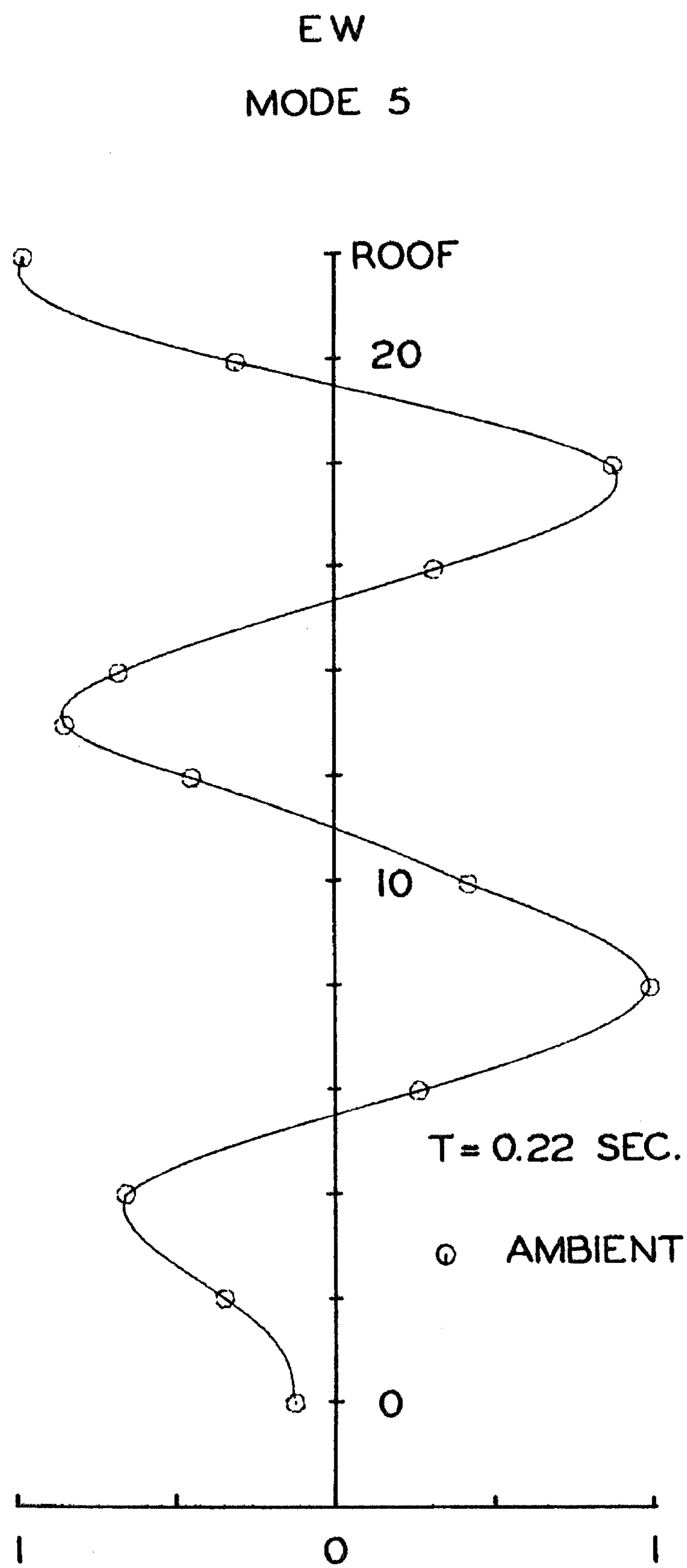

Figure 10e
Fifth EW Mode

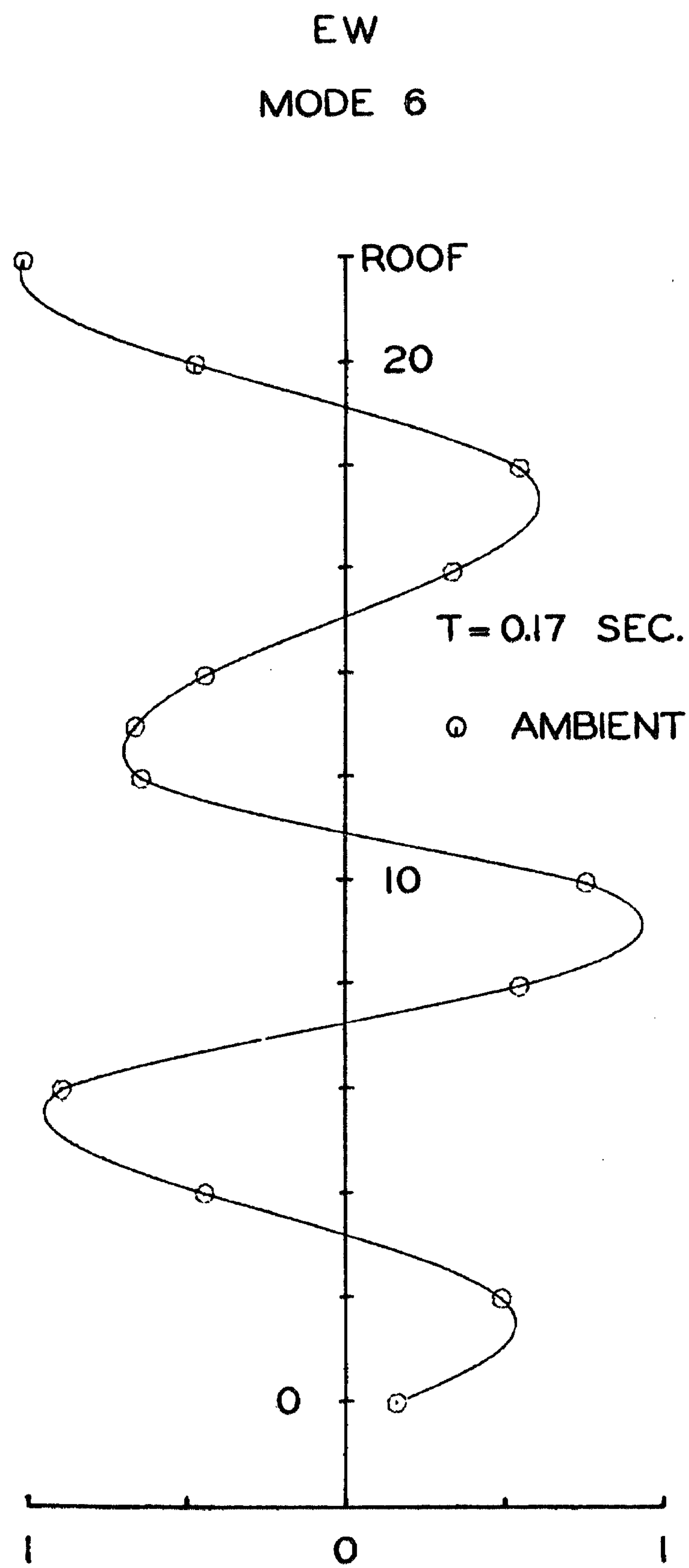

Figure 10f
Sixth EW Mode

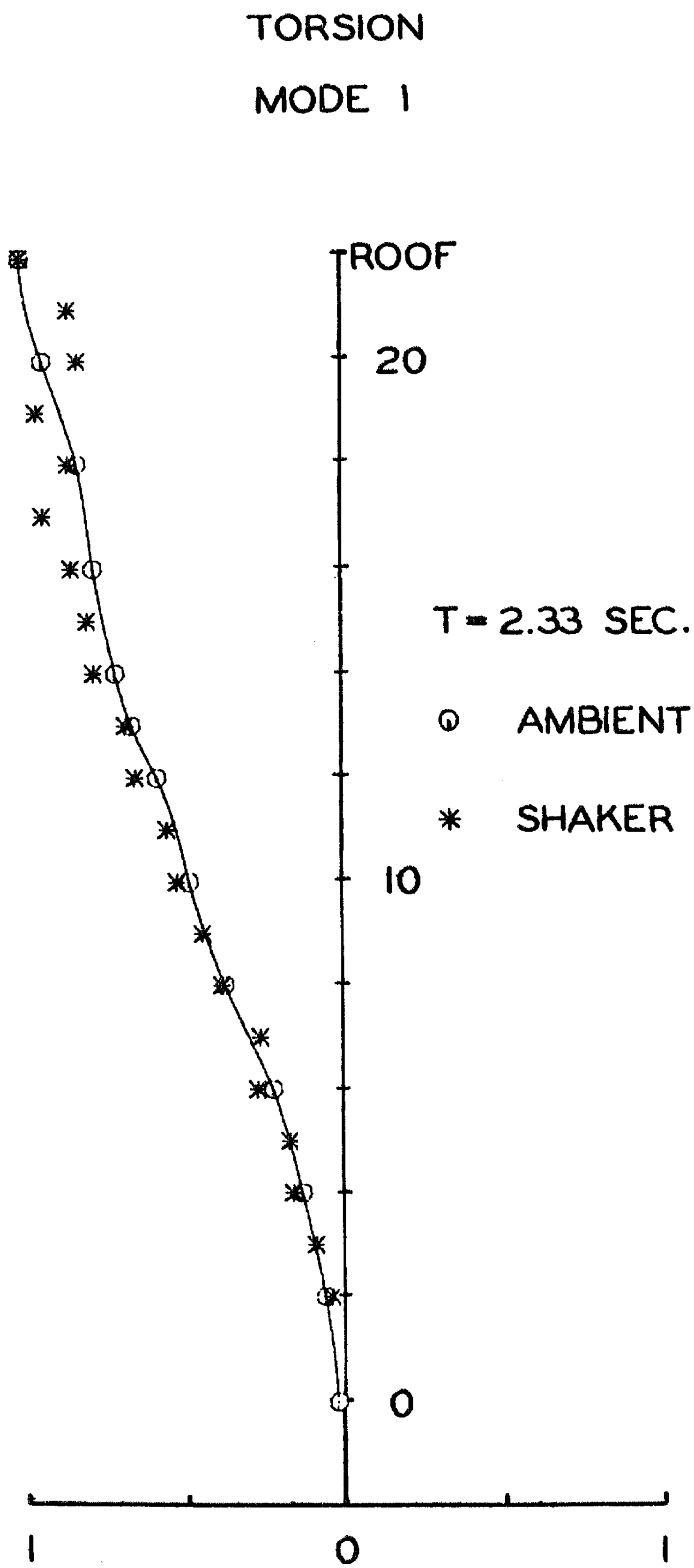

Figure 11a
First Torsional Mode

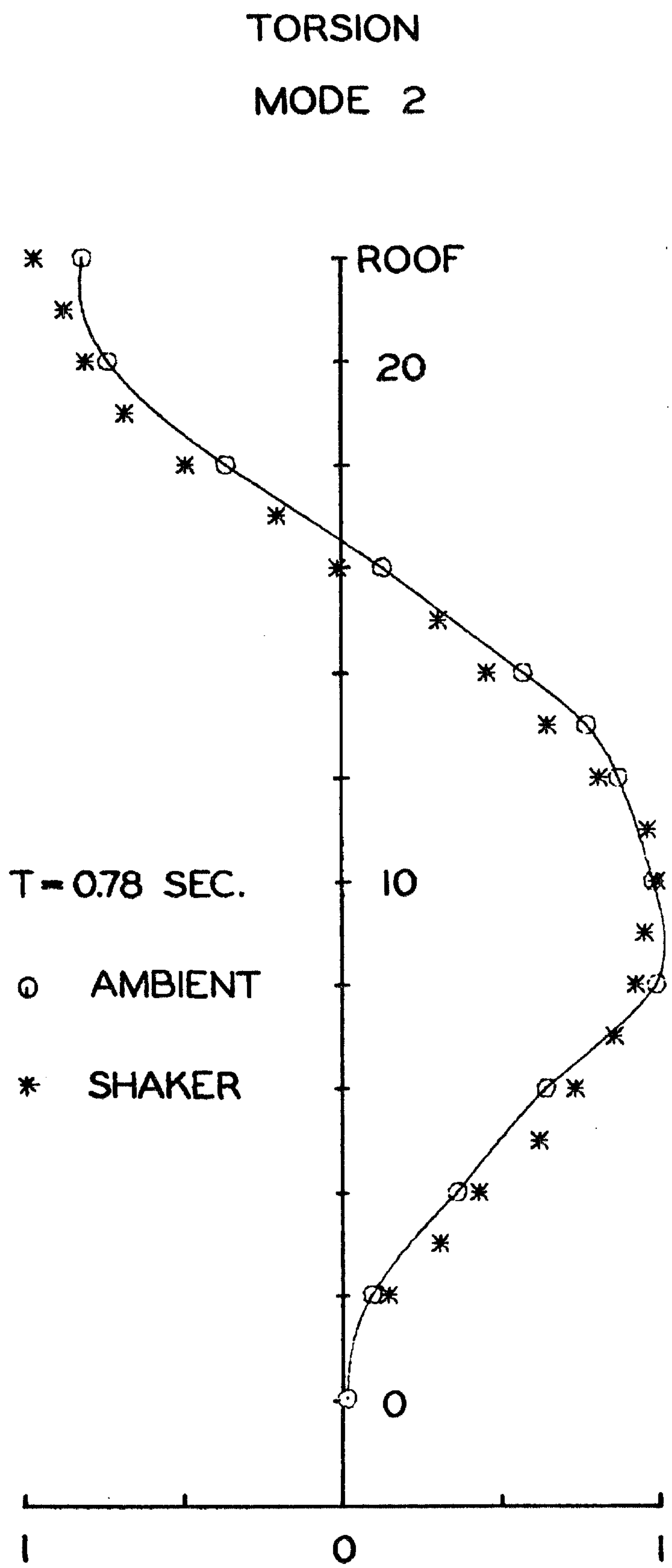

Figure 11b
Second Torsional Mode

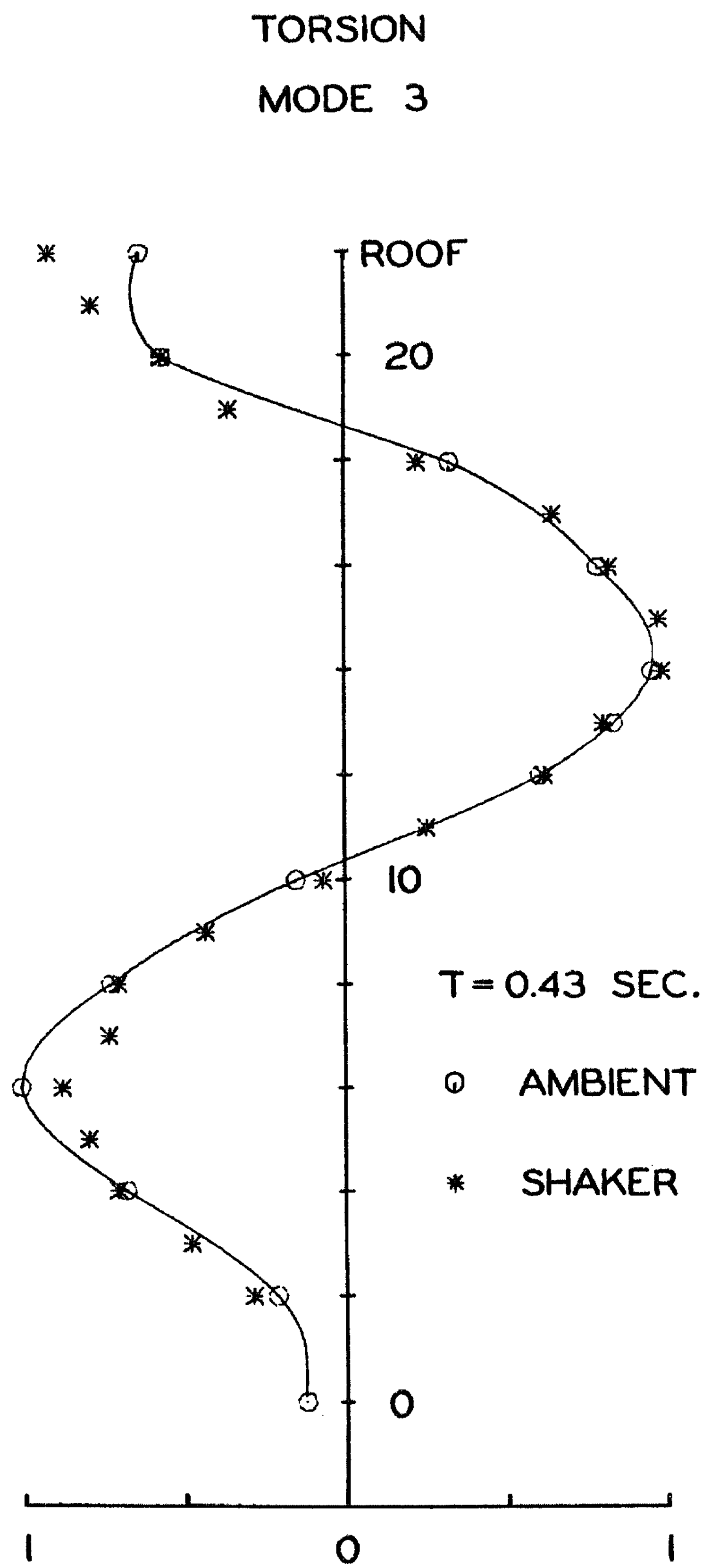

Figure 11c
Third Torsional Mode

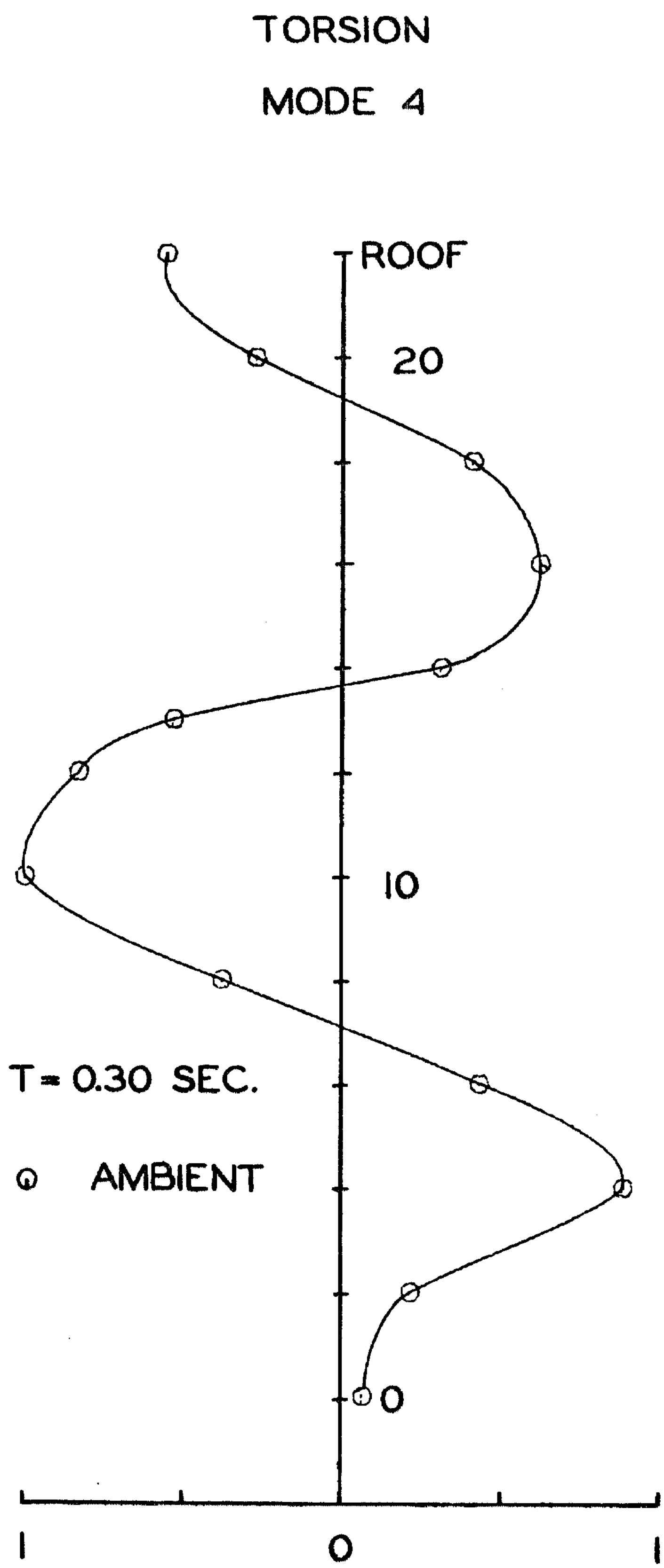

Figure 11d
Fourth Torsional Mode

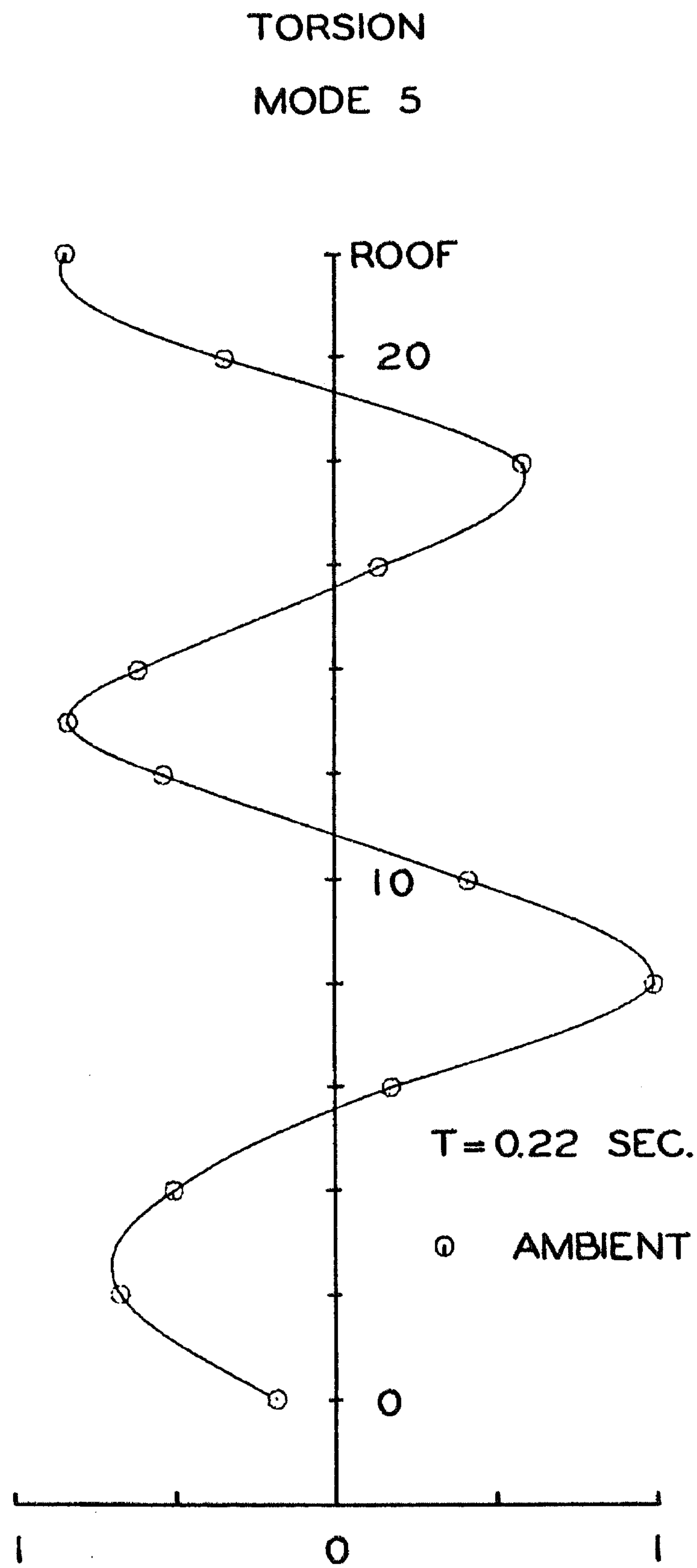

Figure 11e
Fifth Torsional Mode

DAMPING

In the case of sinusoidal steady-state forced vibrations, damping in the building can be determined in several ways; by measuring the peak width at the half-power points, by measuring relative peak amplitude, or by Hudson's method (Keightley et al. 1961). When there is no wind the damping may be determined from a free vibration decay test.

During the ambient vibrations, strictly speaking all these methods fail unless measurements can be taken during the period when either wind or microtremor excitations are random and nearly stationary in time (Ward and Crawford, 1966). During ambient vibration measurements of Gas and Electric Building in San Diego there were only a few instances when such conditions were approximately satisfied. However, even this did not allow straightforward, accurate estimate of damping belonging to each mode. The reason for this was that corresponding frequencies in NS, EW and torsion are closely spaced, thus leading to the spectral overlap in the peak areas. Therefore, for this building, estimates of damping using the half-power method lead to values that are too large. As can be seen from Table 6 damping for the fundamental modes is exceptionally high if one assumes that the values obtained by the vibration experiment are accurate. This is because the EW, NS and torsional frequencies are closest for the fundamental modes of vibration.

More accurate values of damping could be obtained by computing Fourier amplitude spectra with higher resolution, and by the optimum location of seismometers. This was not considered necessary to meet the objectives of this study.

TABLE 6

DAMPING VALUES FOR AMBIENT AND SHAKER EXPERIMENTS

	NS		EW		TORSION	
Mode	Ambient	Shaker*	Ambient	Shaker*	Ambient	Shaker*
1	9.5(5)**	2.6	11.62(10)	1.9	9.2(3)	3.0
2	3.5(5)	2.7	4.0(8)	1.6	5.7(3)	3.4
3	2.6(2)	3.7	4.6(1)	3.1	--	2.9
4	--	3.9	--	2.8	--	3.0
5	--	3.1	--	2.9	--	3.0
6	--	4.4	--	2.8	--	3.4

* Jennings and Hoerner, personal communication

** The numbers in brackets indicate the number of measurements used to obtain the averages which are given in the table.

CONCLUSIONS

The analysis of San Diego Gas and Electric Building vibrations caused by the wind and microtremors lead to the following conclusions:

1. The ambient vibration tests are based on the recording of the displacements many times smaller than vibration tests. However, the analysis indicates that for this building the frequencies and modeshapes of vibration determined from the ambient excitation agree very favorably with the results obtained from the shaker experiment. Therefore, it seems likely that the method of structural testing based on the microtremor and wind induced vibrations can give realistic estimates of the frequencies and modes of vibration for many structures. The building tested is thought to be reasonably typical of modern, multi-story steel frame construction.

2. The determination of an equivalent viscous damping based on the half-power method cannot be used for closely spaced peaks unless the peaks can in some way be separated. This difficulty is caused by spectral overlap and a consequent broadening of the apparent Fourier spectrum peak.

3. The field effort involved in the ambient vibration studies is significantly smaller than for the method using vibration machines. This is because the measuring equipment used for ambient tests is lighter and has fewer components. A group of three to four people required for both ambient and forced vibration experiments can perform necessary measurements for the ambient tests in two to

three days. This time of course depends on the type of the structure. The time necessary to complete forced vibration tests is about five times longer. The total number of necessary measurements in ambient tests is significantly smaller and also each individual measurement requires shorter time intervals. On the other hand, data analysis may be slightly more complicated because it requires the Fourier analysis using digital computers.

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